Another Look at Cosmic Distances

Getting a handle on an expanding — and accelerating — universe. | **By Thomas A. Weil**

IN 1998, ASTRONOMERS ON TWO COMpeting teams announced they had found that not only does our universe appear to be expanding; it's *accelerating* — a conclusion based on observations of supernovae in distant galaxies (*S&T:* September 1998, page 38). This result was bolstered last April when the Hubble Space Telescope glimpsed a very distant supernova and provided strong confirmation of the existence of a universe-pervading repulsive force called the cosmological constant (July issue, page 20).

But how were these results obtained? After all, if an older object is farther away than expected, doesn't that mean that the universe was expanding faster long ago than it is now? Although the acceleration is difficult to explain, this article will attempt to answer these questions. I also provide some software to let you experiment with the parameters that define our universe.

In my previous article, "Looking Back Cosmologically" (*S&T:* September 1997, page 59), I explored the confusing combination of the speed of light and speed of the expanding universe. I presented a small computer program that illustrated the relationship between an object's *distance now* and *distance then*. The program quantified the difference between how far away a galaxy or quasar is today and how far away it was when its light was emitted. In technical books and articles on cosmology, what we call distance then (d_t) is often called the angular-size distance or angular-diameter distance. Distance now (d_n) is often called proper distance, proper-motion distance, comoving distance, transverse comoving distance, or comoving radial distance.

The earlier article did not discuss a third quantity: *luminosity distance* (d_l), which is the apparent distance of an object as determined from its observed brightness and its known inherent luminosity. This quantity, which follows the well-known inverse-square law for how observed brightness decreases with distance, is key in giving us a better understanding of the actual distances and motions in the universe.

How do we know an object's inherent luminosity? In one example, Cepheid variable stars change brightness along a

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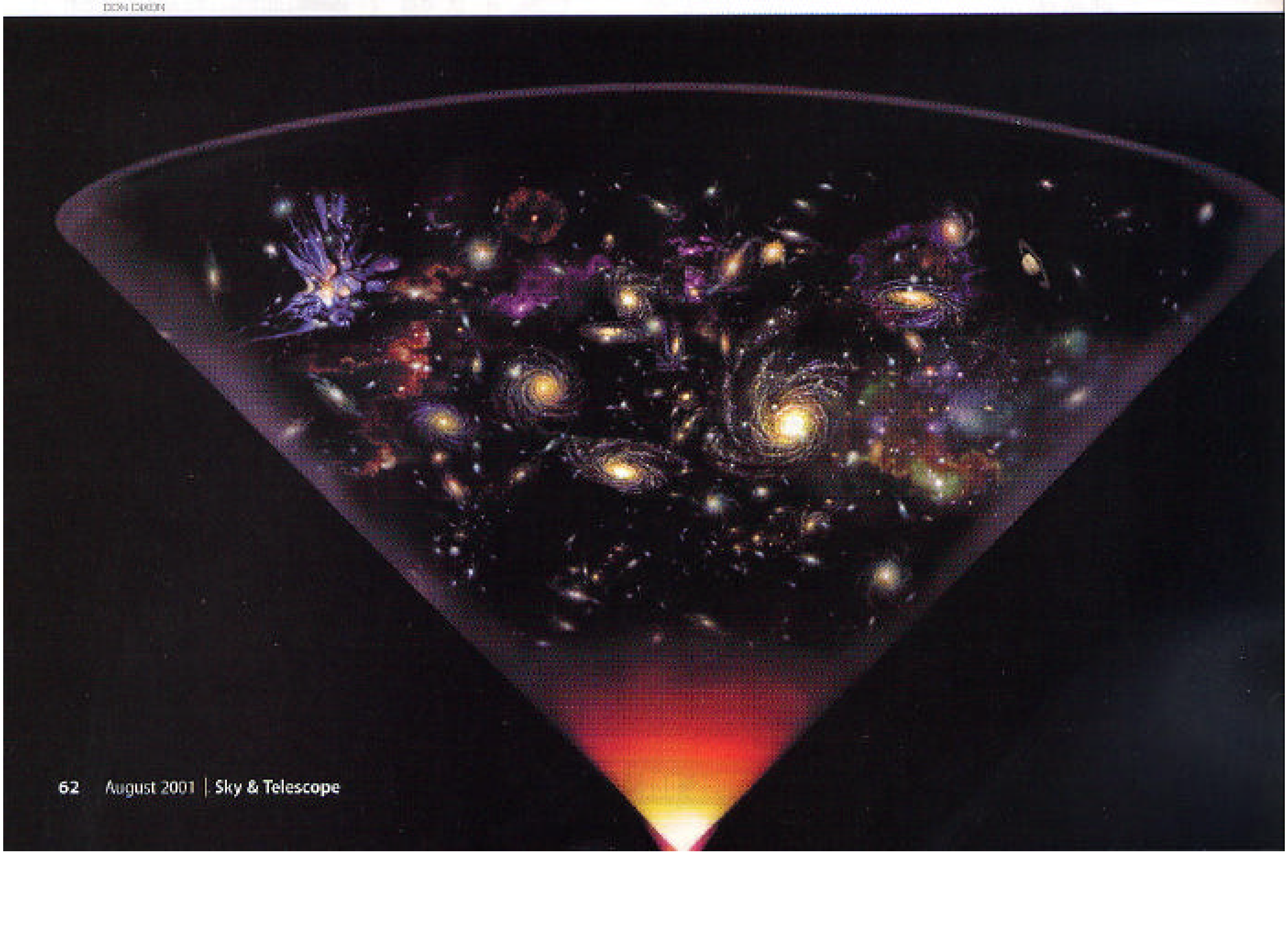


regular cycle whose period is related to the star's luminosity. We can determine the distance of a close-by Cepheid by measuring its parallax. Any star like it will have the same intrinsic brightness; thus, how much dimmer it appears is directly related to how far away it is. (Actually, there are two kinds of Cepheid variables, with different period-luminosity relationships. This confused Edwin Hubble and led him to first estimate the age of the universe as only 2 billion years.)

Similarly, Type Ia supernovae — the kind that occur when white dwarfs explode — have also been used as "standard candles." Astronomers have determined that these exploding stars all reach roughly the same maximum luminosity within 10 percent when differences in brightening and decay rates are taken into account. So when we observe very distant supernovae, we can tell from their measured brightnesses just how far away they must be.

Well, not really. The luminosity distance that we can calculate from measured brightness and known luminosity represents how far away such an object would be in a static universe, one that is neither expanding nor contracting. The expansion of space also affects how bright an object appears, so we have to correct for that if we want to know its true distance. Fortunately, we can measure just how much expansion has occurred since that light was emitted by observing the redshift (z) of the spectrum as we see it now.

Stretched Space

It turns out that d_t, d_n, and d_l are related as follows: $d_n = d_t \times (z + 1)$ and $d_l = d_n \times (z + 1)$. Thus astronomers need only to measure d_l and z to determine all three of these distances.

For a particular object, d_n is larger than d_t because the universe has expanded by a factor of $z + 1$. Furthermore, d_l is larger than d_n because expansion has not only forced the light to travel farther (to the surface of a larger sphere with a radius of d_n), but it has also stretched the light itself as it traveled toward us. Thus, not only do we receive fewer photons per second, but the wavelength of that light has also been stretched (redshifted), which decreases the energy of each photon we receive. As a result, that object now appears to be even farther away than it really is.

For example, a supernova with a redshift of 1.0 actually appears, from its brightness, to lie twice as far away as it really is now. This, in turn, is twice as far away from us as it was when the light we see now was emitted. If we have a 15-billion-year-old "critical-density" universe (defined below), these distances are:

d_t = 6.6 billion light-years,
d_n = 13.2 billion light-years, and
d_l = 26.4 billion light-years.

These differences become even greater for objects with a higher redshift. For $z = 5.0$:

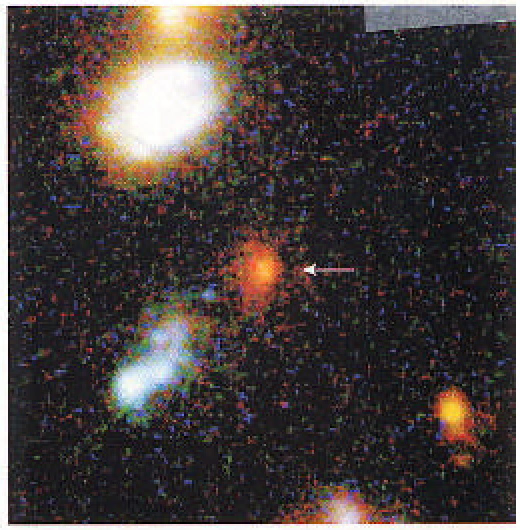
d_t = 4.4 billion light-years,
d_n = 26.6 billion light-years, and
d_l = 159.8 billion light-years.

Graphs of object redshift versus distance, called Hubble diagrams, most commonly use d_l because it can be calculated directly from each object's measured brightness and known luminosity. The diagram below extends the above numbers for our 15-billion-year-old universe, showing a plot of d_t, d_n, and d_l versus redshift.

Only a few years ago, cosmologists quantified our universe by its total mass-energy density, designated by Ω_0. This value was presumed to reflect only the density of matter; other forms of energy were deemed insignificant. In those days, $\Omega_0 = 1$ denoted a "critical" density, which made the universe's expansion slow forever but never fully stop. Such a universe also was "flat," meaning that the corners of a very large triangle (1 billion light-years across) add up to 180°.

It has since become much more complicated. In recent years, astronomers have come to believe the cosmos contains much less matter, such that the matter density of the universe amounts to only 30 percent of the critical density. Yet astronomers still feel strongly that Ω_0 is unity — suggested by inflation theory and supported by recent observations (discussed later in this article). They have been forced to reason that there must be another component making up the difference. Cosmologists now believe this is associated with a repulsive force called the cosmological constant, designated by Λ (Lambda). Thus, Ω_0 has been split into a density for matter (Ω_{matter}) and for the cosmological constant (Ω_Λ).

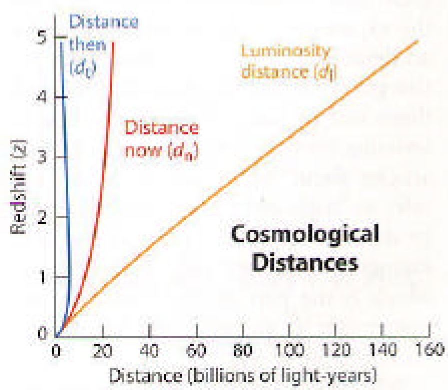
The present-day ratio of an object's speed to its distance is called the *Hubble constant* (H_0), because it appeared to be a fixed number for all the objects at the



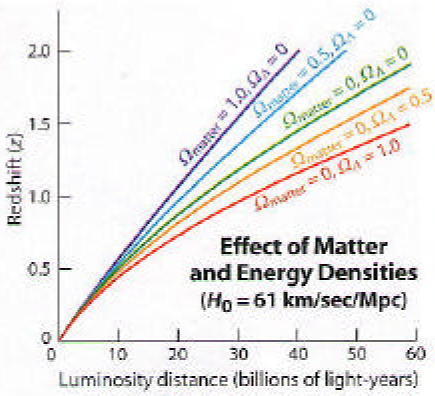
The farthest supernova ever seen has helped pin down some cosmological parameters. The Hubble Space Telescope imaged Supernova 1997ff as part of its northern Hubble Deep Field. The star has a redshift of 1.7, implying that it exploded when the universe was only 4 billion years old. Courtesy NASA and Adam Riess (Space Telescope Science Institute).

moderate distances that Edwin Hubble could measure. However, this value isn't really a constant for two reasons. As I explained in my previous article, that "constant" actually varies with time as the universe expands. The diagram below also reveals that a plot of d_l versus redshift for objects we see now at greater distances is not a straight line but has some curvature, so the Hubble constant thus applies *only* to the initial slope of the curve as it starts from a redshift of zero.

When astronomers plotted their 1998 data for distant supernovae with redshifts near 1.0, their data landed to the right of this curve. This suggested that Λ



A comparison of distances used in this article and the accompanying BASIC program. Each is directly related to an object's redshift.



The expansion rate of the universe is a combination of how much the pull of gravity is slowing receding galaxies (Ω_{matter}) and the degree that space itself is pushing because of an inherent "antigravity" (Ω_Λ). If the sum of these two quantities equals unity, then the universe is "flat" — expansion will slow forever but never fully stop.

was real and that the universe is expanding faster now than it was then. In particular, those astronomers said their measurements showed that $\Omega_{matter} = 0.3$ and $\Omega_\Lambda = 0.7$.

To show how cosmologists reached that conclusion, the diagram above plots d_l versus redshift for universes with $H_0 =$ 61 kilometers per second per megaparsec (the value now preferred by Allan Sandage of the Observatories of the Carnegie Institution of Washington) and several values of Ω_{matter} and Ω_Λ. Although all the curves start from zero with the same initial slope, for decreasing values of Ω_{matter} and for increasing Ω_Λ, the curves pull more to the right. The plot on the opposite page compares the case in which $\Omega_{matter} = 0.3$ and $\Omega_\Lambda = 0.0$ with a model with $\Omega_{matter} = 0.3$ and $\Omega_\Lambda = 0.7$. The rightward bend of the latter indicates why astronomers concluded that the expansion of the universe must be accelerating due to some kind of negative-gravity effect. Boosting Ω_Λ provided them with a better match than simply lowering the values of Ω_{matter}. (Note that articles about these findings sometimes refer to "dark energy" or "vacuum-energy density," which are parts of the cosmological constant, and "dark matter," which is the part of the universe's real matter that we suspect exists but haven't yet been able to see.)

Why would the existence of a cosmological constant make the curve in a Hubble diagram bend farther to the right? If the universe was expanding more slowly in the past than now, it must have taken longer for it to expand by a given factor (and produce a given redshift), so the distant object we see now must have been farther away then, when the light was emitted, for that light to have taken longer to reach us. Thus, for a given redshift and expansion factor since *then*, the distance *now* must also be larger for such objects if the universe had been expanding at a slower rate then.

Why the Rush?

Nevertheless, this acceleration is hard to grasp. Everything else since the instant of the Big Bang is fully consistent with our understanding of physics and relativity; but the repulsive force, or a negative gravity, of the cosmological constant is not. Albert Einstein came up with the idea because pre–Big Bang–theory astronomers told him the universe was static, neither expanding nor collapsing, so he needed to assume the existence of a force that would counteract gravity. When Hubble later found that the universe was indeed expanding, Einstein totally rejected his own idea of a cosmological constant and called it the biggest mistake he ever made. Yet we now see evidence that it *does* exist!

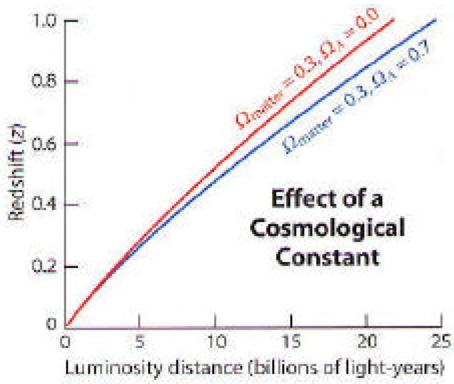
Not only can't current physics explain the existence of a repulsive force acting throughout the universe, but the values of $\Omega_{matter} = 0.3$ and $\Omega_\Lambda = 0.7$ that result from the 1998 supernova data put us in a highly improbable universe. With other values of Ω_{matter} and Ω_Λ, the cosmos would have behaved very differently. That's because even though the force of gravity falls off by the square of the distance, the repulsive force (it is assumed) does not.

On the other hand, if there's no repul-

LOOKBAK2.BAS

```
100 REM  LOOKBAK2.BAS
110 REM  by Thomas A. Weil, taweil@aol.com
120 INPUT "Enter Matter Density of the universe, OmegaM (0 - 2.0)"; OMG
130 IF OMG<0 OR OMG>2 GOTO 120
140 INPUT "Enter Cosmological constant, OmegaL (0. - 1.0)"; LAM
150 IF LAM<0 OR LAM>1 GOTO 140
160 INPUT "Will you enter (A)ge of the universe or (H)ubble constant"; AH$
170 IF AH$="H" OR AH$="h" GOTO 210
180 IF AH$<>"A" AND AH$<>"a" GOTO 160
190 INPUT "Enter Age of the universe NOW in billions of years"; TN
200 TN=TN*1E+09 : HN=60 : M=8 : GOTO 220
210 INPUT "Enter Hubble constant in km/sec/Mpc"; HN : M=1
220 N=200 : DELTA=.0000001 : HFAC=9.7782E+11 : E=2.718282 : PARSEC=3.2616
230 FOR J=1 TO M
240 ORD=.4165/(HN*HN) : OMC=1-OMG-LAM-ORD : Z=5 : AZ=1/(1+Z) : AA=0
250 TRS=0 : TLB=0 : FOR I=1 TO N : A=AZ*(I-.5)/N : AA=AZ+(1-AZ)*(I-.5)/N
260 TRS=TRS+1/SQR(OMC+(OMG/A)+(ORD/(A*A))+(LAM*A*A))
270 TLB=TLB+1/SQR(OMC+(OMG/AA)+(ORD/(AA*AA))+(LAM*AA*AA)) : NEXT I
280 AGEFAC=TLB/N*(1-AZ)+TRS/N*AZ
290 IF AH$="H" OR AH$="h" GOTO 310
300 HN=AGEFAC/TN*HFAC
310 NEXT J
320 TN=AGEFAC/HN*HFAC
330 INPUT "Will you enter (T)ime THEN or (R)edshift of the light we see NOW"; TR$
340 IF TR$="R" OR TR$="r" GOTO 470
350 IF TR$<>"T" AND TR$<>"t" GOTO 330
360 INPUT "Enter Age of the universe THEN, in billions of years"; TTT
370 TTT=TTT*1E+09 : IF TTT>299999 GOTO 410
380 PRINT
390 PRINT " You cannot see back to a time earlier than about 300,000 years"
400 PRINT : GOTO 330
410 PRINT " Finding what redshift matches Age THEN ......" : Z=(TN-TTT)/TN
420 AZ=1/(1+Z) : TRS=0 : FOR I=1 TO N : A=AZ*(I-.5)/N
430 TRS=TRS+1/SQR(OMC+OMG/A+ORD/(A*A)+LAM*A*A) : NEXT I
440 TT=TRS/N*HFAC/HN*AZ : TOL=TOL+DELTA
```

sion at work, how could we explain the Type Ia supernova data? At redshifts approaching 1.0, those supernovae appear only about 25 percent (0.3 magnitude) dimmer, and therefore about 12.5 percent farther away, than values that would fit the Hubble-diagram curve without a cosmological constant. Could cosmic dust possibly have dimmed the light from those distant supernovae as it traveled its long way through space?



The existence of a cosmological constant (Λ) — invented, and then rejected, by Albert Einstein — is now strongly supported by observations of distant supernovae.

Reddened Results?

The members of the High-Z Supernova Search Team and the Supernova Cosmology Project recognized that the dimming might have been caused by cosmic dust. They tried to compensate for this, but their calculations were further complicated because light is also being redshifted as it travels to us. Since this was the first time a "standard candle" had been seen so far away, it was not clear just how precise those corrections were. Furthermore, some cosmic dust may not contain the

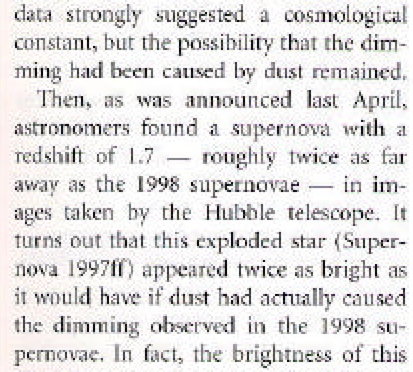
```
450 IF TT/TTT>1-TOL AND TT/TTT<1+TOL GOTO 480
460 Z=Z*(.2+.8*TT/TTT) : GOTO 420
470 INPUT "Enter redshift value for the light we see NOW"; Z
480 DCMR=0 : AZ=1/(1+Z) : FOR I=1 TO N
490 A=AZ+(1-AZ)*(I-.5)/N : ADOT=SQR(OMC+(OMG/A)+(ORD/(A*A))+(LAM*A*A))
500 DCMR=DCMR+1/(A*ADOT) : NEXT I
510 DCMR=(1-AZ)*DCMR/N : X=SQR(ABS(OMC))*DCMR : IF X<=.1 GOTO 550
520 IF OMC>0 GOTO 540
530 RATIO=SIN(X)/X : GOTO 580
540 RATIO=.5*(E^X-E^(-X))/X : GOTO 580
550 Y=X*X : IF OMC<=0 GOTO 570
560 Y=(-Y)
570 RATIO=1+Y/6+Y*Y/120
580 DL=AZ*RATIO*DCMR/(AZ*AZ)*977.82/HN : DN=DL/(1+Z) : DT=DN/(1+Z)
590 AZ=1/(1+Z) : AGE=0 : FOR I=1 TO N
600 A=AZ*(I-.5)/N : AGE=AGE+1/SQR(OMC+OMG/A+ORD/(A*A)+LAM*A*A) : NEXT I
610 TT=HFAC/HN*AZ*AGE/N : TV=TN-TT : DMOD=5*.4343*LOG(DL*1E+09/(10*PARSEC))
620 IF TT<300000 GOTO 380
630 HT=HN*SQR(OMG*(1+Z)^3+(1-OMG-LAM)*(1+Z)^2+LAM)
640 ST=HT*DT/977.82 : SC=Z+1 : SN=HN*DN*1E+09/HFAC
650 PRINT USING "Age Factor NOW  (Age=Fac/HO) = ##.####";AGEFAC
660 PRINT USING "Age of the universe NOW  =####.#### billion years"; TN/1E+09
670 PRINT USING "Age of the universe THEN =####.#### billion years"; TT/1E+09
680 PRINT USING "Light travel time        =####.#### billion years"; TV/1E+09
690 PRINT USING "Redshift of the light we see NOW      =#####.###"; Z
700 PRINT USING "Scale of the universe NOW versus THEN =#####.###"; SC
710 PRINT USING "Distance of object THEN =  #####.### billion light-years"; DT
720 PRINT USING "Distance of object NOW  = ######.### billion light-years"; DN
730 PRINT USING "Luminosity Distance NOW =#######.### billion light-years"; DL
740 PRINT USING "Distance Modulus      = ######.###"; DMOD
750 PRINT USING "Speed away from us THEN =####.### x speed of light*"; ST
760 PRINT USING "Speed away from us NOW  =####.### x speed of light*"; SN
770 PRINT USING "Hubble Parameter THEN   =########.## km/sec/megaparsec"; HT
780 PRINT USING "Hubble Parameter NOW    =########.## km/sec/megaparsec"; HN
790 PRINT " * Not the object's own speed, but caused by the expansion of space."
800 END
```

Only a few years ago, cosmologists quantified our universe by its total mass-energy density, designated by Ω_0. This value was presumed to reflect only the density of matter; other forms of energy were deemed insignificant. A "critical" density of $\Omega_0 = 1$ implied that the universe's expansion slowed forever but never fully stopped.

very fine particles that redden as well as dim starlight (*S&T:* August 1999, page 24), in which case astronomers would not be able to detect dust-generated dimming. In the end, the 1998 supernova data strongly suggested a cosmological constant, but the possibility that the dimming had been caused by dust remained.

Then, as was announced last April, astronomers found a supernova with a redshift of 1.7 — roughly twice as far away as the 1998 supernovae — in images taken by the Hubble telescope. It turns out that this exploded star (Supernova 1997ff) appeared twice as bright as it would have if dust had actually caused the dimming observed in the 1998 supernovae. In fact, the brightness of this new supernova happens to fit almost exactly onto the same $\Omega_{matter} = 0.3$, $\Omega_\Lambda = 0.7$ curve as the 1998 supernovae. Thus, dust clearly was *not* the cause of the dim values. Instead, the new data greatly bolster the case for the existence of a cosmological constant by firmly supporting values of $\Omega_{matter} = 0.35$ and $\Omega_\Lambda = 0.65$.

Strong independent support for this result comes from the latest measurements



The factor used to calculate the age of the universe now from the Hubble constant (H_0) would be directly related to Ω_{matter} if there were no cosmological constant, as shown here. For other cases, including ones with a nonzero Ω_Λ, the rate of expansion has not been a constant power of time. Therefore, the accompanying BASIC program uses multiple integrations to calculate its results.

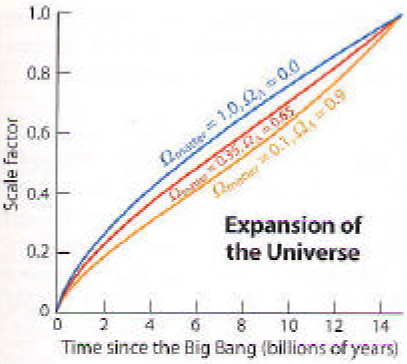
(see page 18) of the wrinkles in the cosmic microwave background (CMB) radiation. Data on the angular spacing of the wrinkles nicely match the value predicted for a flat cosmos but would differ greatly for a curved one. Since astronomers now generally believe that Ω_{matter} is around 0.3 (based on completely independent measurements), the cosmological constant must really exist and must be somewhere around 0.7 to bring the total up to 1.0 to yield a flat universe.

Playing with the Values

My original LOOKBACK.BAS program assumed that the universe expanded as the ⅔ power of time, which applies to a universe with $\Omega_0 = 1$ and no cosmological constant. I have updated and expanded that program to let you specify both Ω_{matter} and Ω_Λ (labeled in the program as OmegaM and OmegaL, respectively). The other inputs are the same as before: specify a value for either the age of the universe or the Hubble constant, as well as either the object's redshift or the Time Then when that light was emitted. The program is called LOOKBAK2.BAS and is provided on the previous two pages, or you can download an annotated version from the Astronomical Software section of *Sky & Telescope's* Web site.

Making a sample run of this program for the new supernova results, using $\Omega_{matter} = 0.35$, $\Omega_\Lambda = 0.65$, $H_0 = 61$, and $z = 1.7$ gives us some valuable information. The program determines that the universe is now 14.8 billion years old and that Supernova 1997ff exploded about 10.8 billion years ago. Because the expansion of the universe initially slowed down while gravity dominated but has been speeding up again since Λ became dominant, that object's velocities away from us then and now just happen to be nearly the same!

It's important to note that the speeds of an object given by this program are not the object's own speed through space, which cannot exceed the speed of light, as described by Einstein's special



This is how a 15-billion-year-old universe would have expanded, given various values of Ω_{matter} and Ω_Λ. Notice how the expansion continues to slow down for the case with $\Omega_\Lambda = 0$, and how it gradually speeds up for other values of Ω_Λ.

theory of relativity. Instead, those speeds are how fast that object was or is being moved away from us by the expansion of space, as described by Einstein's general theory of relativity.

Nevertheless, some cosmologists prefer not to discuss any speeds faster than the speed of light, *c*. Instead, they use the Doppler-shift and special-relativity formula,

$$(1 + z) = \sqrt{(1 + v/c)/(1 - v/c)},$$

to calculate speed, *v*, from redshift. However, this is valid only for very small redshifts (*S&T:* February 1993, page 31). Princeton University cosmologist Jim Peebles says that the expansion of space can indeed move objects away from us faster than the speed of light, and he tells us that an object's speed now is given by the product of today's Hubble constant times the object's distance now, and its speed then is given by the Hubble constant at the time the light began its journey times the object's distance then, as we now use in the new LOOKBAK2.BAS program.

Plug in different values for these cosmological parameters to see the effects. As astronomers make new observations of distant objects, the accepted values could change, possibly altering the age of our universe.

TOM WEIL *(taweil@aol.com) worked as a radar scientist at Raytheon until he retired in 1995. Cosmology has been his hobby for many years.*

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